INTERMEDIATE MICROECONOMICS · PROF. NAVEEN SUNDER

Week 1

Mathematical Review, Basic Economics, Utility, Preferences, and Indifference Curves

"Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses."

— Lionel Robbins, An Essay on the Nature and Significance of Economic Science, 1932

Before You Begin: Quick Self-Assessment

Can you answer these five questions? If any feel unfamiliar, work through Section 1.1 carefully before moving on — the rest of the course builds on these.

1. Solve for x: $3x - 6 = 9$
2. What is the slope of the line $y = 4 - 2x$?
3. If $f(x) = x^3$, what is $\frac{df}{dx}$?
4. Two goods cost \$3 and \$5. You have \$30. Write the equation describing all affordable bundles.
5. If consuming one more apple gives you 10 units of satisfaction and costs \$2, what is your "utility per dollar" from apples?

Answers: (1) $x = 5$ (2) -2 (3) $3x^2$ (4) $3X + 5Y = 30$ (5) 5 utils per dollar

Questions This Chapter Will Answer

- Why do people choose to do anything at all? What drives economic decisions?
- When you skip a lecture to sleep in, what is the true cost of that decision?
- Can we measure how happy a consumer is without asking them to put a number on it?
- Why does consuming more of something eventually feel less satisfying?
- How do we represent a consumer's preferences visually without using numbers?
- Why does a rational consumer always prefer a mix of two goods over extremes?

Chapter Road Map. This chapter builds three tools you will use all semester: **(1) a mathematical toolkit** — slopes, derivatives, and systems of equations; **(2) an economic framework** — opportunity cost, utility, and preferences; and **(3) a graphical language** — indifference curves and the MRS. Each tool builds on the previous one. By the end, you will be able to represent any consumer's preferences on a diagram and quantify how they trade off between goods.

Learning Objectives and Topics

Each objective states what you will be able to *do*. Sections where you learn it are listed beneath.

LO1 Apply the three core differentiation rules and compute slopes of linear and non-linear functions.
§1.1 Mathematical Review

LO2 Define opportunity cost and explain why sunk costs should be ignored in decisions.
§1.2 Basic Economics Concepts

LO3 Explain what utility represents and state the three assumptions on rational preferences.
§1.3 Utility and Preferences

LO4 Draw and interpret indifference curves; explain why they slope downward and bow inward.
§1.4 Indifference Curves

LO5 Define marginal utility and explain the law of diminishing marginal utility.
§1.5 Marginal Utility

LO6 Calculate the MRS from a graph and from a utility function; explain its economic meaning.
§1.6 Marginal Rate of Substitution

Notation Used in This Chapter.

Symbol	Meaning	Symbol	Meaning
X, Y	quantities of goods X and Y	$U(X, Y)$	utility function
$f(x)$	a function of variable x	MU_x	marginal utility of X
df/dx	derivative of f with respect to x	MU_y	marginal utility of Y
$\Delta y/\Delta x$	slope (rise over run)	MRS	marginal rate of substitution
IC	indifference curve	∂	partial derivative symbol

1.1. Mathematical Review

Why does an economics course begin with mathematics? Because every economic trade-off is a rate of change — and slope is how we measure rates of change.

Economics is, at its heart, about understanding relationships—between prices and quantities, between income and consumption, between inputs and outputs. To describe these relationships precisely, we rely on mathematics. This opening section reviews the mathematical tools you will use throughout this course. The goal is not to make you a better mathematician; it is to give you the specific tools needed to study microeconomics rigorously.

We focus on three core topics: (1) the concept of **slope** and how to calculate it for linear functions, (2) **differential calculus** — three rules that let us find slopes of curved functions, and (3) **solving systems of linear equations**. All three will appear again and again as the course progresses.

Mini Case Study: Priya's Study vs. Work Trade-off. Priya is a first-year student deciding how to spend her Saturday. She can study for her economics exam or pick up a 6-hour shift at the campus coffee shop earning \$14/hour. She values free time at \$8/hour. The trade-off Priya faces — where choosing one option means giving up the other — is exactly the kind of relationship economics describes with mathematics. The "slope" of her trade-off tells her how much studying she gives up for each dollar earned. Keep Priya's dilemma in mind: every economic decision involves a rate of exchange between alternatives, and slope is how we measure it.

1.1.1. Functions and Variables

Function. A **function** describes a relationship of dependence between two or more variables. If a variable y depends on a variable x, we write $y = f(x)$ and say "y is a function of x."

The idea of a function is intuitive. The comfort you feel in a room depends on its temperature; your satisfaction from a meal depends on how much food you eat; the revenue a firm earns depends on how many units it sells. In each case, one variable (comfort, satisfaction, revenue) is a function of another (temperature, quantity of food, units sold).

In this course, we will most often deal with functions of two variables. For instance, a consumer's satisfaction—which we will call *utility*—typically depends on how much of good X and good Y they consume:

$$U = f(X, Y).$$

Why This Matters: Functions Are Everywhere in Business. Every time a manager asks "how does profit change if we sell 10 more units?" or "what happens to demand if we raise the price by $1?"—they are asking about a function. The mathematical tools you learn here are used in pricing decisions, financial forecasting, marketing analytics, and operations management.

Check Your Understanding

1. Write a function expressing a firm's revenue R as a function of quantity sold Q and price P. Which variable is dependent? Which are independent?
2. If $U(X,Y) = XY$ and $X = 3$, $Y = 4$, what is U? If X doubles to 6 (with Y fixed), what happens to U?

Now that we know what a function is, we need a way to measure how strongly one variable responds to another. That measure is the slope.

Slope. The **slope** of a function measures the **rate of change** of one variable in response to a change in another. It answers the question: "If x increases by one unit, by how much does y change?"

The simplest setting is a **linear function**, which appears as a straight line on a graph. There are two important cases to keep in mind:

- **Positive slope**: As x increases, y also increases. The line runs upward from left to right.
- **Negative slope**: As x increases, y decreases. The line runs downward from left to right.

Figure 1 illustrates both cases.

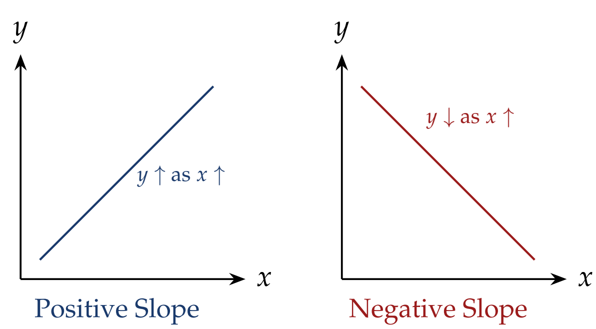


Figure 1. Positive slope (left) versus negative slope (right) for linear functions.

1.1.1.1. Method 1: Rise-Over-Run (Two Known Points)

If you know two points on a line, you can calculate the slope using the **rise-over-run** formula:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Think of this as: "how much does y go up (or down) for each one-unit move to the right along x?"

> **Example.** Suppose a line passes through points $A = (2, 6)$ and $B = (4, 2)$. Calculate the slope.
>
> **Solution.**
> $$\text{Slope} = \frac{2-6}{4-2} = \frac{-4}{2} = -2.$$
> The slope is -2. This means that for every one-unit increase in x, y falls by 2 units. Notice the line is downward-sloping (negative slope), which is consistent with our answer being negative.

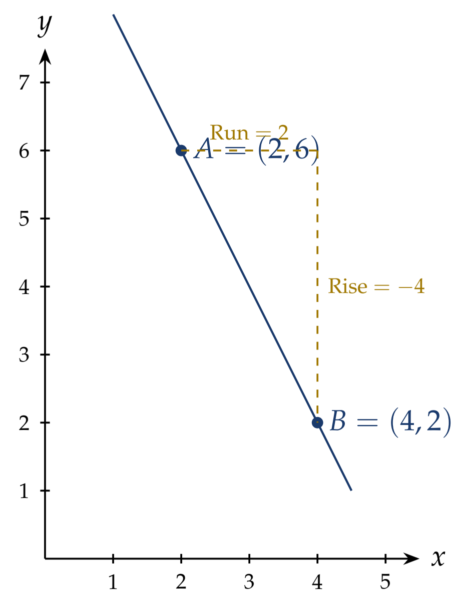


Figure 2. Rise-over-run calculation. Moving from A to B: run $= 2$, rise $= -4$, slope $= -2$.

> **Try It.** A demand schedule shows that when price is \$10, quantity demanded is 80 units, and when price is \$14, quantity demanded is 60 units. Using rise-over-run, compute the slope of the demand curve. What does the sign tell you?

1.1.1.2. Method 2: Slope from the Equation of the Line

Often in economics, you will be given an equation rather than two points. If the equation can be written in the form

$$y = mx + b,$$

then m is the slope and b is the y-intercept. If the equation is not already in this form, rearrange it by isolating y on the left-hand side.

Example. Find the slope of the line $2y + 3x = 6$.

Solution. Isolate y:

$$2y = 6 - 3x$$
$$y = \frac{6}{2} - \frac{3}{2}x = 3 - 1.5x.$$

Comparing to $y = mx + b$, we read off $m = -1.5$. The slope is -1.5.

1.1.1.3. Interpreting Slope: Steeper vs. Flatter

The absolute value of the slope tells you how *responsive* y is to changes in x.

- A **steeper** curve has a larger absolute slope value. A small change in x produces a large change in y.
- A **flatter** curve has a smaller absolute slope value. A large change in x produces only a small change in y.

This distinction will matter greatly when we study demand curves and indifference curves later. A steep demand curve, for instance, means consumers are relatively unresponsive to price changes.

Key Idea: Slope as Responsiveness. The slope is not just a geometric property—it is an economic statement about how strongly one variable responds to another. A steep demand curve says consumers barely change their purchases when price changes. A flat one says they change their purchases dramatically. The same logic applies to indifference curves: the slope at any point tells you how willing a consumer is to trade one good for another.

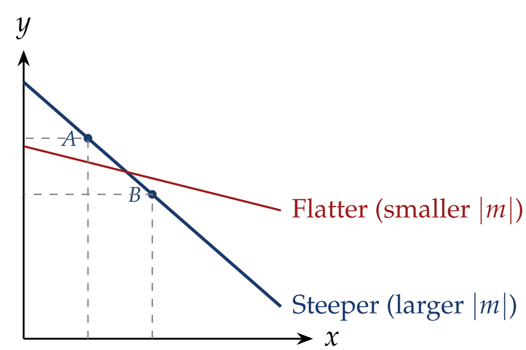


Figure 3. A steeper line has a larger absolute value of slope; y responds more strongly to changes in x.

1.1.2. Slopes of Non-Linear Functions: Differential Calculus

Not all relationships in economics are linear. Indifference curves, cost curves, and production functions are typically curved. To find the slope of a curved function at any given point, we use **differential calculus**.

> **Derivative.** For a function $y = f(x)$, the **first derivative**, written $\frac{dy}{dx}$ or $f'(x)$, gives the slope of the function at every point x.

The key conceptual point is this: the slope of a non-linear function is *not constant*—it differs at each point along the curve. The derivative is the tool that computes the slope at any particular point.

This course requires only three rules of differentiation. Master these three, and you have all the calculus you need for the entire semester.

> **Rule 1 — Constant Rule.** If $y = c$ (any constant), then $\frac{dy}{dx} = 0$.
>
> *In words: A constant never changes, so its rate of change is always zero.*
>
> *Example 1 (trivial):* $y = 7 \implies dy/dx = 0$.
> *Example 2 (course-relevant):* In $U = X^{1/2}Y^{2/3}$, when taking $\partial U/\partial X$, the term $Y^{2/3}$ is a constant — its derivative with respect to X is 0. It stays as a multiplier.

> **Rule 2 — Power Rule.** If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$.
>
> *In words: Bring the exponent down as a multiplier, then reduce the exponent by one.*
>
> *Example 1 (trivial):* $y = x^4 \implies dy/dx = 4x^3$.
> *Example 2 (course-relevant):* $U = X^{1/2} \implies \frac{dU}{dX} = \frac{1}{2}X^{-1/2} = \frac{1}{2\sqrt{X}}$. This is the marginal utility of X — positive but decreasing, capturing diminishing returns.

> **Rule 3 — Sum Rule.** If $y = f(x) + g(x)$, then $\frac{dy}{dx} = \frac{df}{dx} + \frac{dg}{dx}$.
>
> *In words: The derivative of a sum is the sum of the derivatives. Differentiate each term separately.*
>
> *Example 1 (trivial):* $y = x^3 + x^2 \implies dy/dx = 3x^2 + 2x$.
> *Example 2 (course-relevant):* $U = X^2 + Y^{2/3}$. Taking $\partial U/\partial X$: differentiate X^2 to get $2X$; the term $Y^{2/3}$ is a constant (Rule 1), so its derivative is 0. Result: $MU_X = 2X$.

Check Your Understanding

1. Apply the power rule: find dy/dx for (a) $y = x^5$ (b) $y = x^{1/3}$ (c) $y = x^{-2}$.
2. Use Rules 2 and 3 together: find dy/dx for $y = 3x^4 + 5x^{1/2} + 8$.
3. For $U = X^{2/3}Y^{1/3}$, compute $\partial U/\partial X$ (treat Y as a constant using Rule 1).

With the three rules in hand, we are ready to apply them to solve systems of equations — the final mathematical tool before we turn to economics.

The power rule deserves a few worked examples, as it will be used extensively.

Example 1. If $y = x^3$, find $\frac{dy}{dx}$.

Solution. Apply the power rule with $n = 3$:

$$\frac{dy}{dx} = 3x^{3-1} = 3x^2.$$

Example 2. If $y = x^{5/3}$, find $\frac{dy}{dx}$.

Solution. Apply the power rule with $n = 5/3$:

$$\frac{dy}{dx} = \frac{5}{3}x^{5/3-1} = \frac{5}{3}x^{2/3}.$$

Example 3. If $y = \frac{1}{x^{3/4}}$, find $\frac{dy}{dx}$.

Solution. Rewrite using negative exponents: $y = x^{-3/4}$. Now apply the power rule with $n = -3/4$:

$$\frac{dy}{dx} = -\frac{3}{4}x^{-3/4-1} = -\frac{3}{4}x^{-7/4} = -\frac{3}{4x^{7/4}}.$$

Try It. Find $\frac{dy}{dx}$ for each: (a) $y = x^{1/2}$, and (b) $y = 4x^3 + 3x^{1/2} + 10$.

Partial Derivatives. When a function depends on more than one variable—for example, $U = f(X, Y)$—we use **partial derivatives**. The partial derivative $\frac{\partial U}{\partial X}$ measures how U changes when X increases by one unit, *holding Y fixed*. The mechanics are identical to ordinary derivatives: treat Y as a constant and differentiate with respect to X.

Common Mistake: Differentiating the Wrong Variable. When computing $\frac{\partial U}{\partial X}$ for a function like $U = X^2Y^{1/3}$, students sometimes differentiate Y as well. Remember: when taking the partial derivative with respect to X, every term involving only Y is treated as a constant. So $\frac{\partial}{\partial X}(X^2Y^{1/3}) = 2XY^{1/3}$, not $2X \cdot \frac{1}{3}Y^{-2/3}$. The $Y^{1/3}$ rides along as a constant multiplier.

1.1.3. Solving Systems of Linear Equations

Many problems in economics require you to find the values of two unknowns that simultaneously satisfy two equations—for instance, finding the market equilibrium price and quantity where supply equals demand. The standard method is **substitution**.

The Substitution Method:

Step 1. Solve one equation for one variable (e.g., express y in terms of x).

Step 2. Substitute that expression into the other equation. This gives one equation in one unknown.

Step 3. Solve for the remaining unknown.

Step 4. Substitute back to find the other variable.

Example. Solve the following system:

$$4x + 2y = 26 \tag{1}$$
$$x + 5y = 29. \tag{2}$$

Solution.

Step 1. From equation (1), solve for y:

$$2y = 26 - 4x \implies y = 13 - 2x.$$

Step 2. Substitute into equation (2):

$$x + 5(13 - 2x) = 29 \implies x + 65 - 10x = 29 \implies -9x = -36 \implies x = 4.$$

Steps 3–4. Substitute $x = 4$ back:

$$y = 13 - 2(4) = 13 - 8 = 5.$$

Answer: $x = 4$, $y = 5$. You can verify: $4(4) + 2(5) = 26\checkmark$ and $4 + 5(5) = 29\checkmark$.

Key Idea: Systems of Equations in Economics. Virtually every equilibrium problem in this course boils down to solving a system of two equations—a demand equation and a supply equation, or two conditions for consumer optimality. The substitution method you just practiced is the workhorse technique. Get comfortable with it now, and it will serve you throughout the semester.

Check Your Understanding

1. Solve: $2x + y = 10$ and $x + 3y = 15$.
2. A market has demand $Q^d = 100 - 2P$ and supply $Q^s = 20 + 3P$. Set $Q^d = Q^s$ and solve for equilibrium price P^* and quantity Q^*.

The mathematical toolkit is now complete. We turn to the economic ideas that this mathematics will serve — starting with the most fundamental question in the subject: what does it mean to make a rational choice under scarcity?

1.2. Basic Economics Concepts

When a student says attending a "free" lecture costs nothing, what is an economist's response — and why *does it matter for every decision you make?*

Before diving into consumer theory, it is helpful to recall some foundational ideas from introductory economics. These concepts provide the scaffolding on which the more sophisticated analysis of this course rests.

1.2.1. What Is Economics?

A common misconception is that economics is all about money. In fact, economics is much broader: it is the study of how **scarce resources are allocated among alternative uses**. Resources—time, labor, capital, land—are limited. Economics asks how individuals, firms, and societies make decisions about how to use these resources, and whether those decisions lead to good outcomes.

In this course, we focus on **microeconomics**—the behavior of individual consumers and firms. We ask: How does a consumer decide how much to buy? How does a firm decide how much to produce? How do these decisions interact in markets?

> **Why This Matters: Scarcity Is the Fundamental Problem.** Every organization—a hospital allocating nurses, a startup deciding how many engineers to hire, a government setting a budget—faces scarcity. Microeconomics provides a rigorous framework for understanding how these decisions are made and what their consequences are. The tools of consumer and firm theory are directly applied in business strategy, public policy, and competitive analysis.

1.2.2. Opportunity Cost and Sunk Costs

Two cost concepts that students often confuse are opportunity cost and sunk cost. Getting them straight is essential.

> **Opportunity Cost.** The **opportunity cost** of a choice is the value of the next best alternative that was foregone.

A classic illustration: suppose a classmate offers you "free" pizza. Is it really free? Not in an economic sense. To collect the pizza, you might have to walk 15 minutes each way—a 30-minute round trip. Whatever you could have done with that half-hour (study, relax, work) is the opportunity cost of the pizza. Nothing is ever truly free because every choice uses some scarce resource, even if that resource is only time.

> **Sunk Cost.** A **sunk cost** is a cost that has already been incurred and cannot be recovered.

The distinction is best seen through an example.

> **Example: The Movie Ticket.** You pay $15 for a non-refundable movie ticket. You enter the theater, and after 20 minutes you realize the film is dreadful.
>
> - **Sunk cost:** The $15 ticket price. The money is gone whether you stay or leave.
> - **Opportunity cost of staying:** Whatever else you could be doing—watching something at home, studying, sleeping.

The **economically rational** decision is to leave if the opportunity cost of staying exceeds the benefit of the remaining movie. The $15 ticket is irrelevant to this calculation because it cannot be recovered. This is why economists say: "Do not let sunk costs influence forward-looking decisions." In everyday life, people often stay through terrible movies because they "paid for the ticket"—this is a cognitive error known as the *sunk cost fallacy*.

Common Mistake: Confusing Opportunity Cost with Out-of-Pocket Cost. Opportunity cost is not the money you spend—it is the value of what you give up. A student who attends a free tutoring session is not paying tuition, but they are still incurring an opportunity cost equal to the value of their next best use of that time. Conversely, the $15 movie ticket is an out-of-pocket cost that becomes a sunk cost once paid—it should not factor into decisions going forward.

Business Application: Airlines and Sunk Costs. An airline purchases a fleet of planes for $500 million. After delivery, passenger demand collapses due to an economic downturn. The $500 million is now a sunk cost—it cannot be recovered. The airline's forward-looking decision about whether to operate those planes should be based entirely on whether future revenues cover future operating costs (fuel, crew, maintenance). The purchase price is economically irrelevant to that decision. Airlines that factor sunk costs into route decisions make systematically worse choices.

1.2.3. Demand, Supply, and Market Equilibrium

Most students are familiar with demand and supply from introductory economics. We briefly review the key ideas here because they underpin the market analysis that appears throughout the course.

Demand Curve. The demand curve shows the relationship between the price of a good and the quantity consumers wish to buy, holding all other factors constant. It is downward sloping: as price rises, quantity demanded falls ($P \uparrow \Rightarrow Q^d \downarrow$).

Supply Curve. The supply curve shows the relationship between the price of a good and the quantity producers wish to sell, holding all other factors constant. It is upward sloping: as price rises, quantity supplied rises ($P \uparrow \Rightarrow Q^s \uparrow$).

Market Equilibrium. The market equilibrium occurs at the price P^* and quantity Q^* where the quantity demanded equals the quantity supplied:

$$Q^d(P^*) = Q^s(P^*).$$

Think of equilibrium as the "resting point" of the market. At any other price, either buyers or sellers are frustrated, and their actions push the price back toward P^*.

1.2.3.1. What Happens Away from Equilibrium?

When the market is not at equilibrium, forces push it back. Understanding this helps build intuition for how markets work.

- **If $P > P^*$ (price above equilibrium):** At this high price, producers want to sell a lot (quantity supplied is high) while consumers want to buy little (quantity demanded is

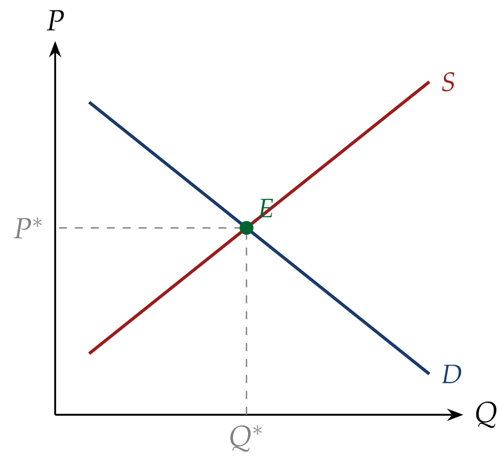


Figure 4. Market equilibrium E where demand meets supply, at equilibrium price P^* and quantity Q^*.

low). The result is **excess supply**—a surplus. Producers, unable to sell their output, will lower prices, pushing the market back toward P^*.

- **If $P < P^*$ (price below equilibrium):** At this low price, consumers want to buy a lot (quantity demanded is high) while producers are unwilling to supply much (quantity supplied is low). The result is **excess demand**—a shortage. Consumers competing for scarce goods will bid prices up, pushing the market back toward P^*.

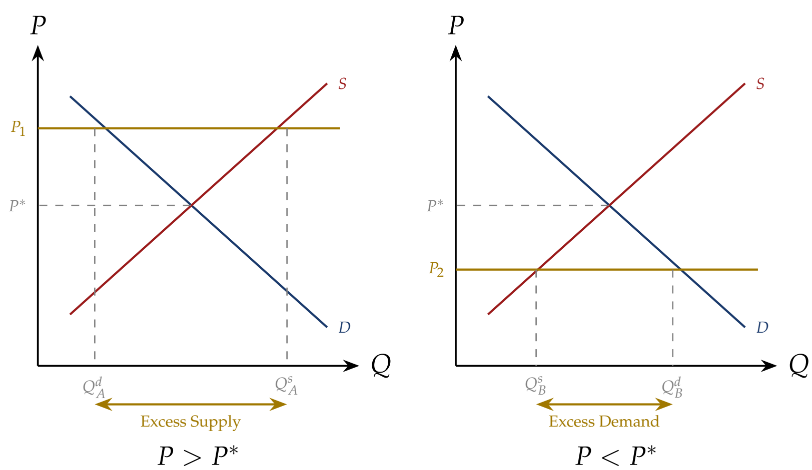
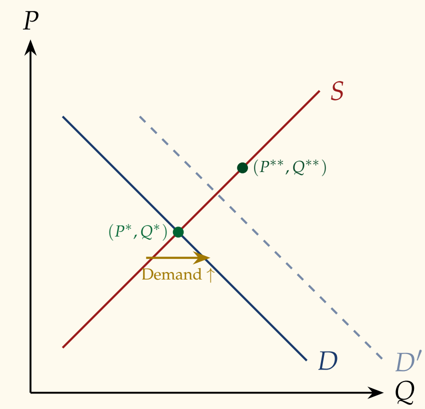


Figure 5. Excess supply when $P > P^*$ (left panel) and excess demand when $P < P^*$ (right panel).

1.2.3.2. Shifts in Demand and Supply: Pandemic Examples

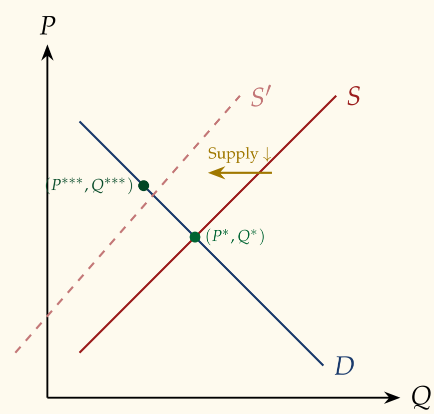
A key distinction in supply-and-demand analysis is between a *movement along* a curve (caused by a price change) and a *shift of* the curve (caused by a change in any factor other than price).

Example 1: Used Cars During the Pandemic. During 2020–2021, demand for used cars increased sharply. This was driven by supply chain disruptions to new-car production, stimulus payments, and consumers avoiding public transit. In the diagram below, demand shifts rightward from D to D'. With supply roughly unchanged, both the equilibrium price and quantity rise: $(P^*, Q^*) \to (P^{**}, Q^{**})$.



This explains why used car prices surged during the pandemic even as more cars changed hands—both price and quantity rose because the shock was on the demand side.

Example 2: Home Furnishings and Supply Chain Delays. During the same period, supply of home furnishings fell due to global supply chain disruptions. Supply shifts leftward from S to S'. Equilibrium price rises while equilibrium quantity falls: $(P^*, Q^*) \to (P^{***}, Q^{***})$.



Practice Question. "Gasoline sells for \$4 per gallon this year and \$3 per gallon last year. But consumers buy *more* of it this year than last year. This is clear proof that the economic theory—that people buy less when price rises—is incorrect." Do you agree?

Answer: No. The demand curve tells us what happens to quantity when *price changes and all else is held equal*. Here, both price and quantity rose—suggesting the *demand curve shifted rightward* between last year and this year (perhaps due to more cars on the road or higher incomes). A rightward demand shift combined with a supply curve that shifted only modestly to the right can produce higher equilibrium price *and* higher equilibrium quantity. There is no contradiction with economic theory.

Why This Matters: Movements vs. Shifts. A very common analytical error—in business reports, journalism, and even policy—is to confuse a *movement along* the demand curve with a *shift* of the demand curve. When prices rise and quantities also rise, that is almost always a signal that something on the demand side has changed, not evidence that demand curves slope upward. Getting this distinction right is essential for interpreting market data correctly.

Check Your Understanding

1. A coffee shop raises prices from $3 to $4 and sells fewer coffees. Is this a movement along the demand curve or a shift? Explain.
2. You paid $200 for a non-refundable concert ticket but feel ill on the night. An economist says you should go *only if* the benefit of attending exceeds what? What should you ignore in this decision?
3. True or false: "The opportunity cost of sleeping in on Saturday morning is zero because you are not spending any money." Justify carefully.

We now have the economic vocabulary for thinking about costs and markets. The next step is to model the consumer directly — how they rank alternatives and what it means to prefer one bundle over another.

1.3. Utility and Preferences

How can an economist represent what a consumer wants without asking them to assign a dollar value to their happiness?

With the mathematical tools in place and the basic market concepts reviewed, we now begin the central task of this first part of the course: understanding how individual consumers make choices. The framework we build here—utility theory and indifference curve analysis—forms the backbone of consumer theory.

1.3.1. What Is Utility?

Utility. **Utility** is the satisfaction or well-being that an individual derives from consuming goods and services.

Utility is expressed as a mathematical function of the goods consumed. In full generality:

$$U = U(X, Y, Z, \ldots)$$

where X, Y, Z, ... represent the quantities of different goods consumed. Because dealing with many goods simultaneously requires advanced mathematics, we simplify in this course by assuming the consumer chooses between only **two goods**, X and Y:

$$U = U(X, Y).$$

This simplification does not sacrifice economic insight. Every lesson we learn with two goods carries over, conceptually, to the case of many goods. The two-good framework is a lens that gives us the same view, with less mathematical complexity.

> **Important: Utility Is Ordinal, Not Cardinal.** Utility is a purely ordinal concept—it ranks bundles from better to worse, but the *numbers* themselves have no inherent meaning. Saying $U = 10$ versus $U = 5$ does not mean the consumer is "twice as happy"; it simply means the first bundle is preferred to the second. You could multiply the entire utility function by 100 and get the same predictions about behavior—only the ranking matters.

> **Why This Matters: Utility in Practice.** While we cannot measure utility directly, firms use closely related concepts all the time. Market research and conjoint analysis ask consumers to rank or choose between product combinations—exactly the ordinal logic of utility theory. Willingness-to-pay surveys and discrete choice models used by tech firms, pharma companies, and automakers all rest on the foundational idea that preferences can be represented by a utility function, even if the numbers themselves are not observable.

1.3.2. Bundles of Goods

Before discussing how consumers choose, we need a precise vocabulary for what they choose *between*.

> **Bundle.** A **bundle** of goods is a specific combination of quantities of two goods. We represent it as a point (X_A, Y_A) in a two-dimensional graph where the horizontal axis measures good X and the vertical axis measures good Y.

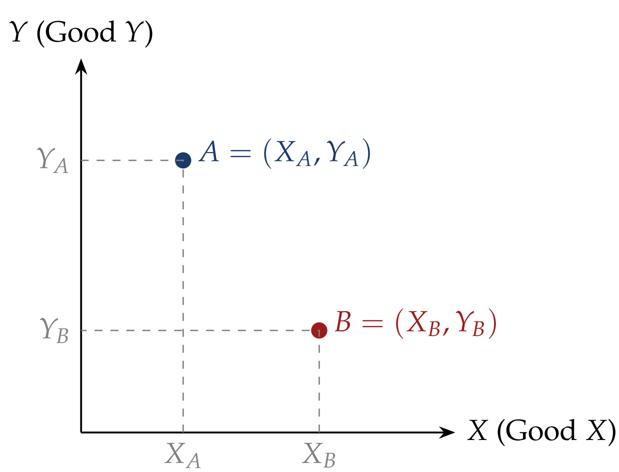


Figure 6. Two bundles, A and B, in the (X, Y) space. Each point represents a specific combination of the two goods.

For example, if the goods are food (X) and clothing (Y), then bundle $A = (2, 6)$ means 2 units of food and 6 units of clothing. Bundle $B = (5, 3)$ means 5 units of food and 3 units of clothing. A consumer must choose between bundles like these. The theory of preferences explains how they do it.

1.3.3. Assumptions on Preferences

To make progress, economists impose three assumptions on how consumers rank bundles. These assumptions are meant to reflect reasonable behavior—you should think critically about whether each one makes sense.

> **Assumption 1 — Completeness.** When a consumer faces any two bundles, A and B, one of three outcomes holds:
>
> (i) A is preferred to B,
>
> (ii) B is preferred to A, or
>
> (iii) the consumer is *indifferent* between A and B (finds them equally desirable).
>
> In other words, "I don't know" is not allowed. A consumer must always be able to rank any two bundles.

> **Assumption 2 — Transitivity.** If a consumer prefers bundle A to bundle B, and bundle B to bundle C, then they must prefer bundle A to bundle C.
>
> Formally: $A \succ B$ and $B \succ C \implies A \succ C$.
>
> Transitivity rules out contradictory rankings. A consumer cannot say "I like pizza more than a burger, and I like a burger more than a salad, but I like a salad more than pizza." Such circular preferences lead to incoherent choices.

> **Assumption 3 — Monotonicity ("More is Better").** If bundle F contains at least as much of both goods as bundle A, and strictly more of at least one good, then F is preferred to A.
>
> Put simply: having more of both goods is always better. A consumer never *dislikes* a good (for now).

> **Key Idea: Why These Three Assumptions?** Completeness ensures consumers can always make a choice. Transitivity ensures their choices are internally consistent—without it, a clever seller could extract unlimited money by cycling the consumer through their own preferences. Monotonicity ensures goods are genuinely desirable. Together, these three assumptions are the minimal conditions needed to represent preferences with a well-behaved utility function.

1.3.3.1. Monotonicity on a Graph

The monotonicity assumption has a clean graphical representation.

The grey regions in Figure 7 reveal a **challenge**: the three assumptions alone do not always allow us to compare all pairs of bundles. For instance, bundle D (which has more Y but less X than A) cannot be ranked directly against A using only monotonicity. Answering how consumers rank such bundles requires a new tool: the indifference curve.

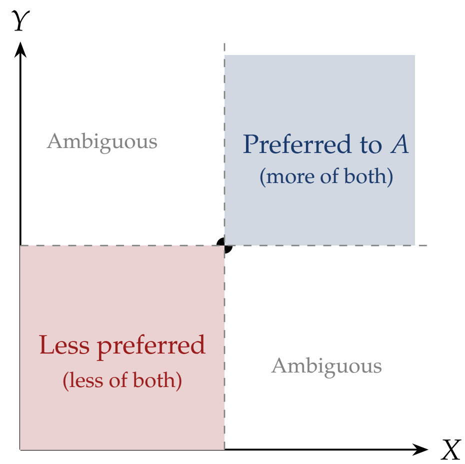


Figure 7. Monotonicity: all bundles in the blue (north-east) region are preferred to A; all bundles in the red (south-west) region are less preferred. Bundles in the grey regions (north-west and south-east) cannot be ranked relative to A using monotonicity alone.

Check Your Understanding

1. A consumer says: "I prefer $A = (3, 2)$ to $B = (2, 3)$, B to $C = (1, 4)$, but C to A." Which assumption is violated?
2. $U(X, Y) = X + Y$. Which bundle gives higher utility: $(3, 5)$ or $(4, 3)$? What does this say about how this consumer values the two goods?
3. Using monotonicity only: can you rank $(5, 3)$ against $(4, 5)$? Why or why not?

The three assumptions tell us that bundles can always be ranked — but they cannot always tell us how much *one is preferred to another, especially when bundles differ in both goods. We need a graphical tool that makes all comparisons possible. That tool is the indifference curve.*

1.4. Indifference Curves

If you gain more coffee but lose some tea, are you better off? The answer depends entirely on your preferences — and the indifference curve is the map that tells us.

> **Indifference Curve.** An **indifference curve** is a curve in (X, Y) space that maps out all bundles that give the consumer the *same level of utility*. A consumer is indifferent between any two bundles that lie on the same indifference curve.

Indifference curves are the workhorse of consumer theory. They let us represent preferences graphically and make comparisons that the three assumptions alone cannot.

1.4.1. Indifference Curves Help Make Comparisons

Return to the challenge from Section 1.3: how do we compare bundle A and bundle D (which has more Y but less X)? Indifference curves provide a bridge. Consider Figure 8.

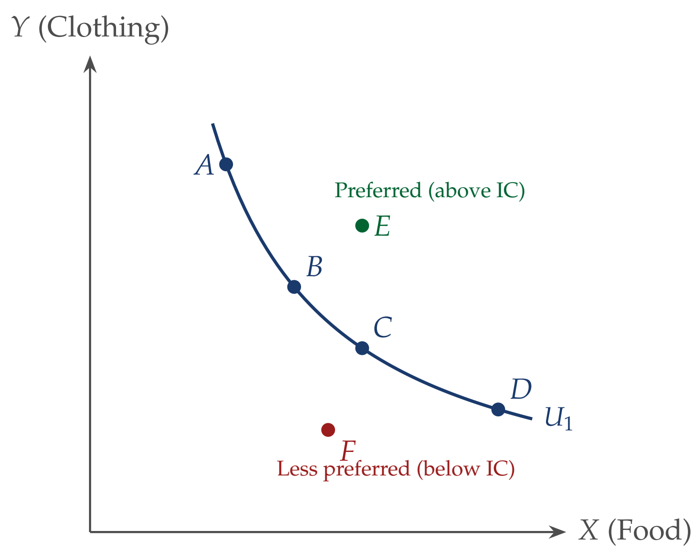


Figure 8. Indifference curve U_1 through bundles A, B, C, D. Focus on three things: (1) all four blue points lie on the same curve — the consumer is equally happy at each; (2) point E is above the curve and preferred (it has more Y than C for the same X, so by monotonicity $E \succ C$, and since $C \sim A$, transitivity gives $E \succ A$); (3) point F is below the curve and less preferred.

Now suppose we want to compare bundle E (above the IC) to bundle A (on the IC). A direct comparison is hard because E has more Y but less X relative to A. But indifference curves make it easy:

1. **Compare E to C:** E lies strictly to the north-east of C (more of both X and Y). By monotonicity, $E \succ C$.
2. **Apply indifference:** C is on the same IC as A, so $C \sim A$ (the consumer is indifferent).
3. **Apply transitivity:** Since $E \succ C$ and $C \sim A$, we conclude $E \succ A$.

Similarly, bundle F (below the IC) is less preferred than any bundle on U_1. This shows how indifference curves cut through the "ambiguous" regions and allow complete comparisons.

> **Key result.** All bundles *above (north-east of)* an indifference curve are preferred to bundles on it. All bundles *below (south-west of)* an indifference curve are less preferred.

1.4.2. The Shape of a Standard Indifference Curve

The typical indifference curve has two important features:

1. **Downward sloping.** If you give up some Y, you must receive more X to remain at the same utility level. This follows from monotonicity: if Y decreases, X must increase to compensate.
2. **Bowed inward toward the origin (convex).** As we move down and to the right along the IC (consuming more X and less Y), the curve becomes flatter. This reflects the *diminishing marginal rate of substitution*, explained in the next section.

1.4.3. Multiple Indifference Curves: Indifference Maps

A consumer has not one but infinitely many indifference curves—one for each utility level. Together they form an **indifference map**. Some key properties of this map:

- **Higher curves are preferred.** A bundle on a higher indifference curve (further from the origin) provides more utility.
- **Indifference curves never cross.** If two ICs crossed, they would imply a contradiction of the transitivity assumption.
- **Every bundle lies on exactly one IC.**

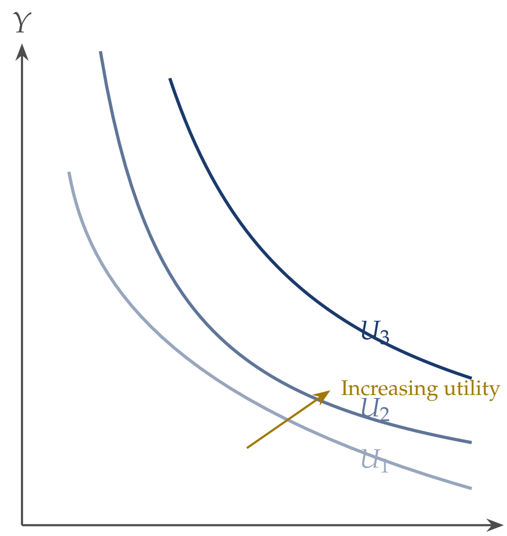


Figure 9. An indifference map: three curves $U_1 < U_2 < U_3$. Focus on two things: (1) curves further from the origin represent higher utility — U_3 is preferred to U_2 which is preferred to U_1; (2) no two curves cross — a crossing would violate transitivity. Every bundle in the diagram lies on exactly one of these curves.

Check Your Understanding

1. For $U = XY$, find the equation of the IC through $(2, 6)$. Does bundle $(3, 4)$ lie on the same IC? What about $(6, 2)$?
2. True or false: "Two indifference curves can cross if one represents a much higher utility level." Prove your answer using transitivity.
3. On a diagram, mark a bundle A and shade: (a) the region preferred to A; (b) the region less preferred. Where does an IC through A lie relative to these regions?

Indifference curves show us the shape of preferences. Now we need a number that measures the strength of preference at any particular point — how much extra utility does one more unit of a good provide? That is marginal utility.

1.5. Marginal Utility

Why does the fifth slice of pizza feel much less satisfying than the first — and what mathematical tool captures this everyday experience?

We have established what utility is and how to represent preferences with indifference curves. We now introduce a concept that connects utility functions to indifference curves mathematically: **marginal utility**.

Marginal Utility. The **marginal utility** of good X, written MU_X, is the change in total utility when the quantity of X consumed increases by one unit, holding the quantity of Y constant:

$$\text{MU}_X = \frac{\partial U}{\partial X} = U'_X.$$

Similarly, $\text{MU}_Y = \dfrac{\partial U}{\partial Y}$.

The partial derivative notation reflects that we hold *all other goods constant* and change only the good in question.

Marginal utility captures the incremental benefit of consuming a little more of a good. Under the monotonicity assumption, marginal utility is always positive: $\text{MU}_X > 0$ and $\text{MU}_Y > 0$.

Key Idea: Marginal Utility Connects Algebra to Graphs. Marginal utility is the bridge between the utility function (algebra) and the indifference curve (geometry). A high MU_X means good X is very valuable at the margin, so the consumer will give up a lot of Y to get more X—which means the indifference curve is steep at that point. The MRS formula in Section 1.6 makes this link precise.

Example: Computing Marginal Utilities.

Let $U = f(X, Y) = X^2 + Y^{2/3}$.

Find MU_X**:**

$$\begin{aligned}\text{MU}_X &= \frac{\partial U}{\partial X} = \frac{\partial}{\partial X}\left(X^2 + Y^{2/3}\right) \\ &= \frac{\partial}{\partial X}(X^2) + \frac{\partial}{\partial X}(Y^{2/3}) \\ &= 2X + 0 = 2X.\end{aligned}$$

(The term $Y^{2/3}$ is treated as a constant when differentiating with respect to X, so its derivative is zero.)

Find MU_Y**:**

$$\begin{aligned}\text{MU}_Y &= \frac{\partial U}{\partial Y} = \frac{\partial}{\partial Y}\left(X^2 + Y^{2/3}\right) \\ &= 0 + \frac{2}{3}Y^{2/3-1} = \frac{2}{3}Y^{-1/3} = \frac{2}{3Y^{1/3}}.\end{aligned}$$

Try It. For $U = X^2 + Y^{2/3}$: evaluate MU_X and MU_Y at the bundle $(X, Y) = (3, 8)$. Which good has higher marginal utility at this bundle?

Additional Example: Cobb-Douglas Utility.

A very common utility function in economics is the **Cobb-Douglas** form:

$$U = X^\alpha Y^\beta, \quad \alpha, \beta > 0.$$

Find MU_X **and** MU_Y**:**

$$\text{MU}_X = \frac{\partial}{\partial X}\left(X^\alpha Y^\beta\right) = \alpha X^{\alpha-1} Y^\beta,$$
$$\text{MU}_Y = \frac{\partial}{\partial Y}\left(X^\alpha Y^\beta\right) = \beta X^\alpha Y^{\beta-1}.$$

For the specific case $U = \sqrt{XY} = X^{1/2}Y^{1/2}$:

$$\text{MU}_X = \frac{1}{2}X^{-1/2}Y^{1/2} = \frac{\sqrt{Y}}{2\sqrt{X}}, \qquad \text{MU}_Y = \frac{1}{2}X^{1/2}Y^{-1/2} = \frac{\sqrt{X}}{2\sqrt{Y}}.$$

Business Application: Marginal Value in Platform Decisions. Streaming services use the logic of marginal utility when expanding content libraries. Adding the 10th movie in a genre a user loves produces much less satisfaction than adding the first in a genre they have never seen. This is diminishing marginal utility in action — it tells platforms that *variety* often adds more value than depth, shaping both recommendation algorithms and content acquisition budgets.

Check Your Understanding

1. For $U = X^{1/2}Y^{1/2}$: compute MU_X and MU_Y. Are they positive? What happens to MU_X as X increases (holding Y fixed)?
2. A consumer has $MU_X = 10$ and $MU_Y = 5$. Which good provides more utility at the margin? If the consumer could costlessly get one more unit of either good, which should they choose?
3. True or false: "If $MU_X = 0$, the consumer has reached maximum possible utility from good X." Explain.

Going deeper: For students who want to explore further: the concept of diminishing marginal utility underpins the theory of risk aversion and insurance. If your utility of wealth is concave (diminishing MU), you will pay to avoid risk — even when the expected value is the same. This is covered in advanced microeconomics.

Marginal utility measures how much you value the next unit of a single good. But in reality, you always choose between goods — giving up some of one to get more of another. The marginal rate of substitution captures exactly that trade-off.

1.6. Marginal Rate of Substitution (MRS)

You are at a buffet with both pizza and pasta. How many slices of pasta would you give up for one more slice of pizza — and why does that number change as you eat more pizza?

We now turn to one of the most important concepts in consumer theory: the **marginal rate of substitution**. The MRS connects the shape of the indifference curve to the consumer's preferences.

Marginal Rate of Substitution (MRS). The **MRS** is the amount of good Y that the consumer is willing to give up in exchange for one additional unit of good X, while remaining on the same indifference curve (i.e., keeping utility constant).

Equivalently, the MRS equals the **slope of the indifference curve** at any given point:

$$\text{MRS} = \text{slope of IC} = \left.\frac{\Delta Y}{\Delta X}\right|_{U=\text{const}}.$$

The MRS is typically *negative*—to get more X, you must give up some Y—and we usually refer to its absolute value.

1.6.1. Calculating MRS from a Graph

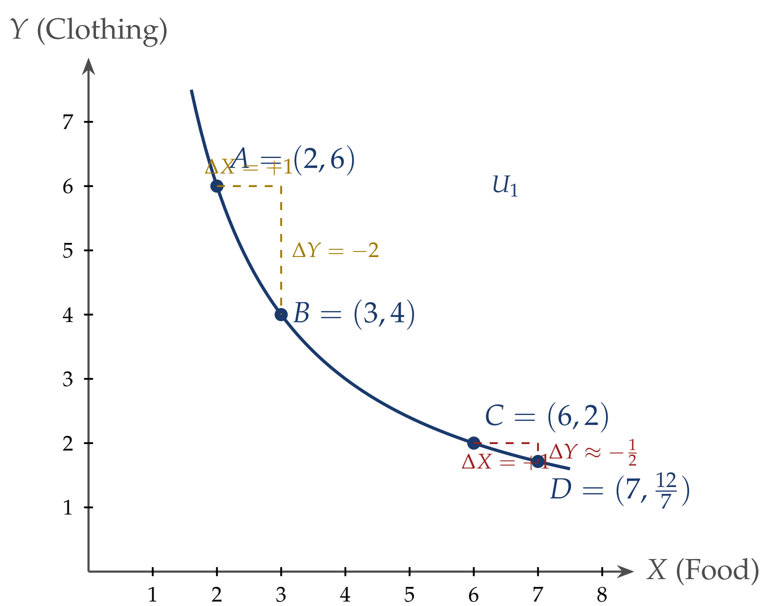


Figure 10. Diminishing MRS along U_1 (where $U_1 = 12$). Compare the two dashed triangles carefully. Near A, moving one unit right ($\Delta X = +1$) requires giving up *two* units of clothing ($\Delta Y = -2$) — a steep trade-off. Near C, the same one-unit gain in food only costs half a unit of clothing ($\Delta Y \approx -\frac{1}{2}$). This shrinking sacrifice is diminishing MRS, and it is why the curve flattens as you move right.

From Figure 1.6.1:

$$\text{MRS}_{AB} = \frac{\Delta Y}{\Delta X} = \frac{4-6}{3-2} = \frac{-2}{1} = -2.$$

To gain one more unit of food (moving from A to B), the consumer gives up 2 units of clothing.

Further along the curve, moving from C to D:

$$|\text{MRS}_{CD}| \approx \frac{1}{2}.$$

Now the consumer gives up only $\frac{1}{2}$ a unit of clothing to gain one more unit of food.

1.6.2. Diminishing MRS

Notice that the MRS fell from 2 (at A) to $\frac{1}{2}$ (at C). This is not a coincidence—it reflects a fundamental property of typical preferences:

> **Diminishing MRS.** As a consumer acquires more of good X (moving right along an IC), the MRS (in absolute value) decreases. The consumer becomes less and less willing to give up Y for additional X.

Intuition: When you have very little food and lots of clothing, food is precious to you and you are willing to give up a lot of clothing for one more meal. But as you accumulate more food, each additional unit of food matters less, and you are only willing to give up a small amount of clothing for it. This is the same logic as *diminishing marginal utility*—the more you have of something, the less valuable an additional unit becomes.

Geometrically, diminishing MRS is what makes indifference curves **bowed inward (convex) toward the origin**. The flatter the IC gets as you move right, the smaller the absolute value of its slope, which is the MRS.

Why This Matters: Diminishing MRS and Product Bundling. Diminishing MRS is why variety has value. At a restaurant, the first appetizer is delicious, but by the fifth you would gladly trade several appetizers for one dessert. This is why restaurants offer set menus with variety, and why subscription boxes, streaming services, and buffets all try to offer a mix of goods rather than more and more of the same thing. The economic intuition behind diminishing MRS shapes real product and pricing decisions.

1.6.3. Balanced Consumption Is Preferred

A direct implication of diminishing MRS (and convex ICs) is that **balanced bundles are preferred to extreme ones**.

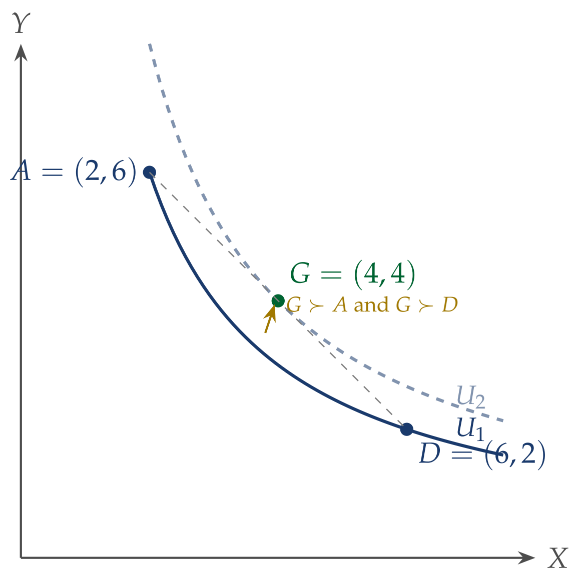


Figure 11. Balanced bundle $G = (4, 4)$ is preferred to the extremes $A = (2, 6)$ and $D = (6, 2)$. The dashed line connects A and D — their midpoint G lies *above* U_1 (on U_2), confirming that the consumer prefers the balanced mix. This is a direct consequence of the IC bowing inward: convexity means averages beat extremes.

As Figure 11 shows, the average bundle $G = (4, 4)$ lies above the indifference curve U_1 that passes through the extreme bundles $A = (2, 6)$ and $D = (6, 2)$. Since G is above U_1, it yields higher utility—$G \succ A$ and $G \succ D$. Consumers prefer balance.

1.6.4. Calculating MRS Using Marginal Utilities

There is an elegant and practical formula that connects MRS to the marginal utilities we computed in Section 5:

MRS–MU Formula.

$$\text{MRS} = -\frac{\text{MU}_X}{\text{MU}_Y}.$$

The MRS equals the negative ratio of marginal utilities.

The intuition is this: MU_X measures how much utility you gain from one more unit of X, and MU_Y measures how much utility you lose from giving up one unit of Y. To stay on the same indifference curve (keep utility constant), the gain and loss must cancel—hence the ratio.

Derivation (intuition): Along an indifference curve, total utility is constant: $dU = 0$. By the chain rule:

$$dU = \text{MU}_X \cdot dX + \text{MU}_Y \cdot dY = 0 \implies \frac{dY}{dX} = -\frac{\text{MU}_X}{\text{MU}_Y}.$$

Since $\text{MRS} = \frac{dY}{dX}$ along the IC, we get the formula above.

Example: MRS for $U = X^2 + Y^{2/3}$.

We computed earlier that $\text{MU}_X = 2X$ and $\text{MU}_Y = \frac{2}{3Y^{1/3}}$. Therefore:

$$\text{MRS} = -\frac{\text{MU}_X}{\text{MU}_Y} = -\frac{2X}{\frac{2}{3Y^{1/3}}} = -\frac{2X \cdot 3Y^{1/3}}{2} = -3XY^{1/3}.$$

At the bundle $(X, Y) = (2, 8)$, for instance:

$$\text{MRS} = -3(2)(8)^{1/3} = -3 \cdot 2 \cdot 2 = -12.$$

The consumer would give up 12 units of Y for one more unit of X at this bundle.

Additional Example: MRS for Cobb-Douglas Utility.

Let $U = X^\alpha Y^\beta$. Then $\text{MU}_X = \alpha X^{\alpha-1} Y^\beta$ and $\text{MU}_Y = \beta X^\alpha Y^{\beta-1}$.

$$\text{MRS} = -\frac{\text{MU}_X}{\text{MU}_Y} = -\frac{\alpha X^{\alpha-1} Y^\beta}{\beta X^\alpha Y^{\beta-1}} = -\frac{\alpha}{\beta} \cdot \frac{Y}{X}.$$

For $U = \sqrt{XY}$ ($\alpha = \beta = 1/2$):

$$\text{MRS} = -\frac{Y}{X}.$$

This is a clean and memorable result: the MRS for a symmetric Cobb-Douglas utility function equals minus the ratio of goods consumed.

Try It. For $U = \sqrt{XY}$, compute the MRS at the bundle $(3, 12)$ and at $(6, 6)$. Confirm that the MRS diminishes in absolute value as X increases.

Common Mistake: Forgetting the Negative Sign in MRS. The formula $\text{MRS} = -\text{MU}_X/\text{MU}_Y$ always yields a negative number for normal goods (since both MUs are positive). The IC slopes downward. Students sometimes drop the negative sign and then get confused when comparing MRS magnitudes. When comparing the "steepness" of substitution, always use $|\text{MRS}|$ to avoid sign confusion.

Check Your Understanding

1. For $U = X^{1/2}Y^{1/2}$, compute MRS as a function of X and Y. Evaluate at $(2, 8)$ and at $(4, 4)$. Is MRS diminishing?
2. A consumer has $|\text{MRS}| = 4$ at the current bundle. The market price ratio $P_X/P_Y = 2$. Should they buy more X or more Y? *(Hint: compare the internal trade-off rate to the market rate.)* ⋆ *Preview of Week 2.*
3. True or false: "If the MRS is constant along an indifference curve, the IC is a straight line." Explain.

Going deeper: For stronger students: the MRS equals the ratio of partial derivatives — a result from multivariate calculus called the implicit function theorem applied to the level curve $U(X, Y) = \bar{U}$. You will see this technique again whenever you need to find the slope of any curve defined implicitly by an equation.

We have now built all three tools: the mathematical toolkit, the economic framework, and the graphical language of indifference curves and MRS. Before reviewing the key terms, take a moment to see how the practice questions translate these tools into exam-ready skills.

Where Does This Take Us?

- **Week 2** introduces the budget constraint and combines it with indifference curves to find the consumer's optimal bundle. The MRS you calculated here becomes the key ingredient in the tangency condition.
- The **MRS–MU formula** ($\text{MRS} = -MU_X/MU_Y$) reappears in Week 2 as the left-hand side of the optimality condition $P_X/P_Y = MU_X/MU_Y$.
- **Diminishing MRS** guarantees that the consumer's optimal bundle is unique — without convexity, multiple optima could exist.
- **Production theory (Week 6)** uses an identical apparatus: isoquants play the role of indifference curves, and the marginal rate of technical substitution mirrors the MRS exactly.

Five Common Exam Mistakes — and How to Avoid Them

1. **Including sunk costs in forward-looking decisions.** Once a cost is sunk, it is gone. Ask only: what are the future costs and benefits? The money already spent is irrelevant.
2. **Confusing a movement along a curve with a shift of the curve.** A price change causes a movement along demand or supply. A change in income, tastes, or input costs causes a shift. These are fundamentally different events.
3. **Treating utility numbers as cardinal.** $U = 10$ is not "twice as good" as $U = 5$. Only the ranking matters — you could replace every utility number with any order-preserving transformation and get the same predictions.
4. **Dropping the negative sign in the MRS.** $\text{MRS} = -MU_X/MU_Y$ is negative. The indifference curve slopes downward. Always work with $|\text{MRS}|$ when comparing magnitudes.

5. **Applying the power rule without rewriting negative exponents.** $1/x^2 = x^{-2}$ before differentiating. Forgetting to rewrite first is the single most common algebraic error on calculus questions.

1.7. Key Terms

Function
: A mathematical relationship where one variable depends on another; written $y = f(x)$. *Used throughout the course.*

Slope
: Rate of change of y per unit change in x; equals $\Delta y/\Delta x$ for linear functions, or the derivative for non-linear ones.

Derivative
: The slope of a function at any point; computed using the constant, power, and sum rules.

Partial Derivative
: Derivative with respect to one variable, holding all others fixed. Written $\partial U/\partial X$.

Opportunity Cost
: Value of the best foregone alternative when a choice is made. *Used throughout the course.*

Sunk Cost
: A cost already incurred and unrecoverable; should not affect forward-looking decisions.

Market Equilibrium
: Price–quantity pair where $Q^d = Q^s$.

Excess Supply
: $Q^s > Q^d$; arises when price exceeds equilibrium.

Excess Demand
: $Q^d > Q^s$; arises when price is below equilibrium.

Utility
: Satisfaction from consuming goods; ordinal (ranking), not cardinal (magnitude). *Used throughout the course.*

Bundle
: A specific combination (X, Y) of two goods.

Completeness
: Every two bundles can be ranked or declared indifferent.

Transitivity
: $A \succ B$ and $B \succ C$ implies $A \succ C$.

Monotonicity
: More of both goods is always preferred ("more is better").

Indifference Curve (IC)
: All bundles yielding the same utility. *Used throughout the course.*

Indifference Map
: The full set of ICs; one per utility level.

Marginal Utility (MU)
: Extra utility from one more unit of a good; $MU_X = \partial U/\partial X$. *Used throughout the course.*

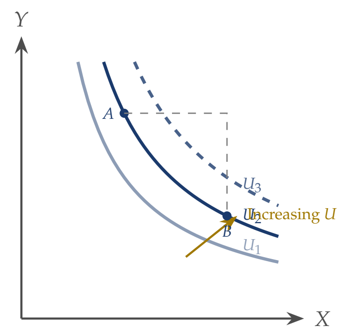
Marginal Rate of Substitution (MRS)
: Units of Y willingly given up per unit of X gained, staying on the same IC; equals $-MU_X/MU_Y$. *Used throughout the course.*

Diminishing MRS
: |MRS| falls as X rises along an IC; produces convex (bowed-in) ICs. *Used throughout the course.*

Chapter at a Glance

Your one-page revision guide — everything you need to review before an exam or tutorial.

Standard Indifference Curve



Key Formulas

Slope (linear)	$\Delta y/\Delta x$	p. 3
Power rule	$d(x^n)/dx = nx^{n-1}$	p. 3
Sum rule	$d(f+g)/dx = f' + g'$	p. 3
Marginal utility	$MU_X = \partial U/\partial X$	p. 20
MRS formula	$\text{MRS} = -MU_X/MU_Y$	p. 22
MRS (Cobb-Douglas)	$\text{MRS} = -(\alpha/\beta)(Y/X)$	p. 22

Three Assumptions on Preferences

- **Completeness** — any two bundles can always be ranked
- **Transitivity** — preferences are logically consistent
- **Monotonicity** — more of both goods is always better

Properties of Indifference Curves

- Downward sloping (from monotonicity)
- Convex / bowed inward (diminishing MRS)
- Higher curves = higher utility
- Two ICs never cross (from transitivity)

The Three Differentiation Rules

Rule	Formula	Example
Constant rule	$d(c)/dx = 0$	$d(7)/dx = 0$
Power rule	$d(x^n)/dx = nx^{n-1}$	$d(x^3)/dx = 3x^2$
Sum rule	$d(f+g)/dx = f' + g'$	$d(x^2+x)/dx = 2x+1$

Five Key Terms

1. **Opportunity cost** — value of best alternative foregone
2. **Utility** — ordinal ranking of satisfaction
3. **Indifference curve** — equal-utility bundles
4. **Marginal utility** — extra utility per extra unit
5. **MRS** — consumer's internal trade-off rate

Learning Objectives Covered

- LO1 — three differentiation rules ✓
- LO2 — opportunity and sunk costs ✓
- LO3 — utility and preference assumptions ✓
- LO4 — indifference curves and their properties ✓
- LO5 — marginal utility and diminishing MU ✓
- LO6 — MRS from graph and formula ✓

The thread connecting it all: Calculus gives us the tools → opportunity cost frames every decision → utility represents preferences → indifference curves draw them → MRS quantifies the trade-off. Week 2 adds the budget constraint and finds where the consumer chooses to be.

Recommended Videos

Four short videos selected for clarity and direct relevance to this chapter's content. Watch them in order for best results.

1. **Opportunity Cost** *(MRU — Marginal Revolution University)* ~5 min
https://www.youtube.com/watch?v=MnIDhRKGGbo
One of the clearest short explanations of opportunity cost available, produced by Tyler Cowen and Alex Tabarrok. Directly reinforces Section 1.2 and is particularly useful before attempting the sunk-cost and opportunity-cost questions in Level 1.

2. **Calculus: Power Rule (Introduction)** *(Khan Academy)* ~9 min
https://www.youtube.com/watch?v=l9cP9K8fDr4
A step-by-step walkthrough of the power rule with multiple examples, including fractional exponents. Closely mirrors the Rule 2 box in Section 1.1 and is ideal for students who want extra practice before the derivatives questions in Level 1–2.

3. **Indifference Curves and Utility** *(Jacob Clifford, ACDC Economics)* ~9 min
https://www.youtube.com/watch?v=V7cSn87b3YE
Covers utility, the shape of indifference curves, and why they bow inward. One of the most-watched introductory treatments on YouTube; directly reinforces Sections 1.3–1.5 and is excellent preparation for the Week 2 material.

4. **Marginal Rate of Substitution** *(Economics in Many Lessons)* ~10 min
https://www.youtube.com/watch?v=7Qtr9AQBEPA
Derives the MRS both graphically and algebraically using the $-MU_X/MU_Y$ formula, with Cobb-Douglas examples that mirror the worked examples in Section 1.6. Recommended before attempting Level 2–3 questions on MRS.

1.8. Practice Questions

How to use these questions. Questions are arranged in three levels. Level 1 builds vocabulary and computational fluency; Level 2 requires analysis and diagram work; Level 3 demands synthesis across topics. Difficulty: ★ easy ★★ intermediate ★★★ challenging. Full solutions follow in a separate document. Attempt every question independently first.

Level 1: Conceptual and Foundational

★ **[Slopes of Linear Functions]**

(a) Find the slope of the line passing through $(1, 7)$ and $(4, 1)$.

(b) Find the slope of the line $3y - 6x = 12$.

(c) Without calculating, explain in one sentence why the slope you found in (b) is positive.

★ **[Power Rule]** Find the first derivative of each function:

(a) $y = x^4$

(b) $y = x^{2/5}$

(c) $y = \dfrac{1}{x^{1/2}}$

(d) $y = 5x^3 + 2x^{1/2} + 7$

★ **[Partial Derivatives]** Let $U(X, Y) = X^{1/3}Y^{2/3}$.

(a) Compute MU_X.

(b) Compute MU_Y.

(c) Are both marginal utilities positive? Which preference assumption does this reflect?

★ **[Opportunity Cost]** A student attends a 3-hour review session (no tuition charge). During those hours she could have earned $15/hour at her part-time job. She values leisure at $10/hour.

(a) What is the opportunity cost of attending the session?

(b) Is the session truly "free"? Explain.

(c) How does this illustrate opportunity cost?

★ **[Sunk Costs]** A restaurant owner spent $50,000 on renovations six months ago. Business has been poor since. Her accountant says the renovations are "sunk." Explain what "sunk" means and why the $50,000 should not affect the decision of whether to keep operating.

★ **[Demand and Supply]** For each scenario, identify whether the demand or supply curve shifts (and in which direction), and state what happens to equilibrium price and quantity.

(a) A drought destroys a large share of the coffee crop.

(b) Consumer incomes rise, increasing demand for restaurant meals.

(c) New brewing technology lowers the cost of producing craft beer.

★ **[Preference Assumptions]** For each statement, identify which assumption (completeness, transitivity, or monotonicity) is violated, if any.

(a) The consumer prefers A to B, B to C, and C to A.

(b) When choosing between D and E, the consumer says "I cannot decide."

(c) A consumer prefers fewer apples and fewer oranges to more of both.

(d) A consumer prefers bundle $F = (3, 5)$ to $G = (4, 6)$.

★ **[Explain to a Friend — Level 1]** *No algebra allowed.* Explain in 3–4 plain-English sentences why indifference curves slope downward. Your explanation should be convincing to a friend who has never studied economics.

Level 2: Intermediate Analysis

★★ **[Solving Linear Equations]** Solve each system:

(a) $3x + y = 15$ and $x + 4y = 16$

(b) $5x - 2y = 8$ and $x + y = 4$

★★ **[Indifference Curves and Monotonicity]** $U = XY$.

(a) Find the IC through $(4, 5)$.

(b) Find the IC through $(2, 10)$.

(c) Are these the same curve? What does this tell you about these two bundles?

(d) Which bundle gives higher utility? Is this consistent with monotonicity?

★★ **[Marginal Utility]** $U = 3X + Y^{1/2}$.

(a) Compute MU_X and MU_Y.

(b) Evaluate both at bundle $(2, 9)$.

(c) As Y increases (holding X fixed), what happens to MU_Y? What preference property does this reflect?

★★ **[MRS from Graph]** Moving along an IC from $P = (1, 10)$ to $Q = (3, 6)$ to $R = (6, 4)$:

(a) Calculate the average MRS between P and Q.

(b) Calculate the average MRS between Q and R.

(c) Is MRS diminishing? What does this imply about IC shape?

(d) Draw a labelled diagram showing P, Q, R on a convex IC. Indicate the rise and run for the PQ calculation.

★★ **[MRS Formula]** Compute $\text{MRS} = -\text{MU}_X/\text{MU}_Y$ for each:

(a) $U = X^{3/4}Y^{1/4}$

(b) $U = X^{1/2} + Y^{1/2}$

(c) $U = \ln X + 2\ln Y \quad (\partial \ln X/\partial X = 1/X)$

(d) $U = 2X + 3Y$ (Is MRS constant? What type of goods are X and Y?) ⋆ *Revisit after Week 2*

★★ **[Indifference Curves: Transitivity and Crossing]** Two ICs $U_1 < U_2$ cross at bundle Z.

(a) Let A be on U_1 only and B on U_2 only. State the implied preference ordering of A, B, and Z.

(b) Show that a crossing violates transitivity.

(c) Conclude why ICs can never cross.

★★ **[Excess Demand and Supply]** Demand: $Q^d = 100 - 2P$; Supply: $Q^s = 10 + 3P$.

(a) Find equilibrium P^* and Q^*.

(b) At $P = 15$: is there a surplus or shortage? By how much?

(c) At $P = 25$: is there a surplus or shortage? By how much?

(d) Draw a labelled diagram marking equilibrium and both off-equilibrium prices.

★★ **[Data-Based: Bundle Ranking]** The table below shows five consumption bundles for a consumer whose utility function is $U(X,Y) = XY$.

Bundle	X	Y	$U = XY$
A	2	8	?
B	4	4	?
C	1	12	?
D	6	2	?
E	3	6	?

(a) Compute utility for each bundle and rank them from highest to lowest.

(b) Which bundles lie on the same indifference curve? Explain why.

(c) A student claims: "Bundle C is better than A because it has more Y." Is she correct? Explain carefully using the utility values.

(d) Which bundle exhibits the most "balanced" consumption? Compute the MRS at bundle B and interpret it.

(e) ⋆ *Revisit after Week 2:* If prices are $P_X = \$3$ and $P_Y = \$2$ and income $I = \$20$, which of these bundles are affordable? Which is optimal?

★★ **[Explain to a Friend — Level 2]** A friend says: "Utility numbers are like test scores — if I get 80 utils from apples and 40 utils from oranges, apples make me exactly twice as happy." Write a short paragraph correcting this misunderstanding. Explain the difference between ordinal and cardinal utility in plain English.

Level 3: Advanced Problems

★★★ **[Full MRS Analysis]** $U(X,Y) = X^{2/3}Y^{1/3}$.

(a) Compute MU_X and MU_Y.

(b) Compute MRS as a function of X and Y.

(c) Evaluate MRS at $(3,6)$ and $(6,3)$. Compare and explain the economic meaning.

(d) Find the slope of the IC through $(3,6)$ and verify it matches MRS.

(e) As the consumer moves along the IC from $(3,6)$ toward more X and less Y, what happens to MRS? Draw and label the IC.

★★★ **[Properties of Indifference Curves]** Consumer has well-behaved preferences.

(a) True or False: "A steeper IC at a given bundle means the consumer values Y relatively more than X." Justify carefully.

(b) True or False: "An IC can slope upward if the consumer dislikes one good." Explain with an example.

(c) A consumer is indifferent between $(1,8)$ and $(4,2)$. If $U = X^aY^{1-a}$, find a.

★★★ **[Demand, Supply, and Simultaneous Shifts]** $Q^d = 500 - 2P$; $Q^s = -100 + 3P$.

(a) Find equilibrium.

(b) Supply shifts to $Q^s = -160 + 3P$ (rising fuel costs). Find new equilibrium.

(c) Simultaneously, demand shifts to $Q^d = 560 - 2P$ (new travel trend). Find equilibrium with both shifts.

(d) Draw a fully labelled diagram showing original and new equilibria.

★★★ **[Utility and MRS: Non-Standard Form]** $U(X,Y) = XY + 2X$.

(a) Compute MU_X and MU_Y.

(b) Compute MRS. Is it constant?

(c) Find bundles on $U = 20$ where $X = 2$ and $X = 4$.

(d) Compute MRS at each bundle and verify it is diminishing.

★★★ **[Conceptual Essay: MRS and Market Trade-offs]** ⋆ *Revisit after Week 2.*

(a) Explain in well-reasoned prose (not bullet points) why a convex IC reflects diminishing MRS and why this implies balanced bundles are preferred to extreme ones.

(b) The MRS is the consumer's internal trade-off rate. How does it relate to the market trade-off rate (relative prices)? Why does this comparison matter?

(c) A consumer has MRS $= -3$ at the current bundle. The price ratio $P_X/P_Y = 2$. Is the consumer at an optimum? What should they do?

★★★ **[Calculus and MU: Production Analogy]** $F(L, K) = L^{1/2}K^{1/2}$.

(a) Compute $MP_L = \partial F/\partial L$.

(b) Compute $MP_K = \partial F/\partial K$.

(c) Show that as L increases (holding K fixed), MP_L decreases. Economic meaning?

(d) Compute MRTS $= -MP_L/MP_K$. Interpret economically.

(e) Evaluate MRTS at $(4, 9)$ and $(9, 4)$. Compare and explain. ⋆ *Connects to production theory in Week 6.*

★★★ **[Comprehensive: Utility, ICs, and MRS]** $U(X, Y) = (X + 1)(Y + 2)$.

(a) Compute MU_X and MU_Y.

(b) Compute MRS as a function of X and Y.

(c) Find the equation of the IC for $U = 30$.

(d) Find bundles on this IC where $X = 0$, $X = 2$, $X = 4$.

(e) Compute MRS at each bundle and verify it is diminishing.

(f) Sketch the IC, labelling all three points and the approximate slope at each.

(g) At the $X = 2$ bundle, what does the MRS tell you about the consumer's valuation?

★★★ **[Opportunity Cost in Business Decisions]** A software engineer earns \$120,000/year. She is considering starting her own company: \$50,000 upfront costs, \$90,000 first-year revenue.

(a) What is the economic profit of starting the company, accounting for opportunity cost?

(b) Based on economic profit alone, should she start it?

(c) Name two factors not captured in economic profit that might still influence her decision.

★★★ **[Explain to a Friend — Level 3]** Your friend is about to take his first economics exam. He has memorised the definition of MRS but does not understand what it means. Write a short explanation (4–6 sentences) that uses a concrete everyday example — no jargon, no formulas — to convey the economic intuition of the MRS and why diminishing MRS makes sense.

Full solutions follow in the solutions document. Attempt all questions independently before consulting them.

Solutions to Practice Questions

> **Note on numbering.** Q1–Q8 are Level 1; Q9–Q17 are Level 2; Q18–Q26 are Level 3. Solutions to the new questions Q8, Q17, and Q26 are added at the end of their respective levels.

Solution to Q1

(a) Using the rise-over-run formula with $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 1)$:

$$\text{slope} = \frac{1-7}{4-1} = \frac{-6}{3} = -2.$$

(b) Rearrange $3y - 6x = 12$:

$$3y = 6x + 12 \implies y = 2x + 4.$$

The slope is $m = 2$.

(c) In (b), as x increases by 1, y increases by 2. Both variables move in the same direction, so the slope is positive. Equivalently, the equation $y = 2x + 4$ has a positive coefficient on x.

Solution to Q2

Apply the power rule $\frac{d}{dx}x^n = nx^{n-1}$ and the constant rule.

(a) $y = x^4$: $\frac{dy}{dx} = 4x^3$.

(b) $y = x^{2/5}$: $\frac{dy}{dx} = \frac{2}{5}x^{2/5-1} = \frac{2}{5}x^{-3/5}$.

(c) $y = x^{-1/2}$: $\frac{dy}{dx} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x^{3/2}}$.

(d) By the addition rule: $\frac{dy}{dx} = 5 \cdot 3x^2 + 2 \cdot \frac{1}{2}x^{-1/2} + 0 = 15x^2 + x^{-1/2} = 15x^2 + \frac{1}{\sqrt{x}}$.

Solution to Q3

$U = X^{1/3}Y^{2/3}$.

(a) $\text{MU}_X = \frac{\partial U}{\partial X} = \frac{1}{3}X^{-2/3}Y^{2/3}$.

(b) $\text{MU}_Y = \frac{\partial U}{\partial Y} = \frac{2}{3}X^{1/3}Y^{-1/3}$.

(c) Both $\text{MU}_X > 0$ and $\text{MU}_Y > 0$ for positive X and Y. This reflects the **monotonicity** assumption: more of either good always increases utility.

Solution to Q4

(a) The student's next best use of 3 hours is working at $15/hour, earning $3 \times 15 = \$45$. This is the opportunity cost.

(b) No. Even though no money is paid, the student sacrifices $45 of potential income. Nothing is truly free when it uses scarce time.

(c) Opportunity cost captures the value of the best foregone alternative—here, the foregone wages. Even a zero-price activity is not free in economic terms.

Solution to Q5

"Sunk" means the $50,000 has already been spent and cannot be recovered regardless of the restaurant owner's future actions. Since the $50,000 is gone whether she stays open or closes, it provides no information about the profitability of continuing operations. The relevant decision depends on whether future revenues exceed future costs. Factoring in the $50,000 would be the sunk cost fallacy—allowing a past, unrecoverable expense to distort a forward-looking choice.

Solution to Q6

(a) A drought reduces coffee supply. Supply shifts left. Equilibrium: P^* rises, Q^* falls.

(b) Higher incomes increase demand for restaurant meals. Demand shifts right. Equilibrium: P^* rises, Q^* rises.

(c) New technology lowers production costs, increasing supply. Supply shifts right. Equilibrium: P^* falls, Q^* rises.

Solution to Q7

(a) Violates **transitivity**: $A \succ B$, $B \succ C$, yet $C \succ A$ creates a cycle. Transitivity requires $A \succ C$.

(b) Violates **completeness**: the consumer must be able to rank D and E (or declare indifference). "I cannot decide" is not admissible.

(c) Violates **monotonicity**: fewer of both goods is less preferred, not more preferred.

(d) Violates **monotonicity**: bundle $G = (4, 6)$ has strictly more of both goods than $F = (3, 5)$, so G should be preferred. Here the consumer prefers F to G, which contradicts monotonicity.

Solution to Q8 (Explain to a Friend — Level 1)

A good answer conveys the following ideas in plain English. An indifference curve shows all the bundles a consumer is equally happy with. If we move to a bundle with more of good Y (holding X fixed), the consumer is happier — so to keep them equally happy, we must take away some X. That means as Y goes up, X must go down to stay on the same curve. Any curve where one variable goes up while the other goes down is downward sloping. No algebra required: it follows directly from the fact that both goods are desirable.

Solution to Q9 (formerly Q8)

(a) System: $3x + y = 15$ and $x + 4y = 16$. From equation 1: $y = 15 - 3x$. Substitute into equation 2:

$$x + 4(15 - 3x) = 16 \implies x + 60 - 12x = 16 \implies -11x = -44 \implies x = 4.$$

Then $y = 15 - 12 = 3$. Answer: $(x, y) = (4, 3)$. Check: $3(4) + 3 = 15\checkmark$; $4 + 12 = 16\checkmark$.

(b) System: $5x - 2y = 8$ and $x + y = 4$. From equation 2: $y = 4 - x$. Substitute into equation 1:

$$5x - 2(4 - x) = 8 \implies 5x - 8 + 2x = 8 \implies 7x = 16 \implies x = \frac{16}{7}.$$

Then $y = 4 - \frac{16}{7} = \frac{12}{7}$. Answer: $(x, y) = \left(\frac{16}{7}, \frac{12}{7}\right)$.

Solution to Q9

(a) At $(4, 5)$: $U = 4 \times 5 = 20$. IC: $XY = 20$, or $Y = 20/X$.

(b) At $(2, 10)$: $U = 2 \times 10 = 20$. IC: $XY = 20$, or $Y = 20/X$.

(c) They are the same curve ($U = 20$). Both bundles yield the same utility, so the consumer is indifferent between $(4, 5)$ and $(2, 10)$.

(d) Both give $U = 20$. They are on the same IC, so neither is preferred—the consumer is indifferent. This is consistent with monotonicity: a bundle is preferred only if it has strictly more of at least one good and no less of the other.

Solution to Q10

$U = 3X + Y^{1/2}$.

(a) $\text{MU}_X = \frac{\partial U}{\partial X} = 3$. $\text{MU}_Y = \frac{\partial U}{\partial Y} = \frac{1}{2}Y^{-1/2} = \frac{1}{2\sqrt{Y}}$.

(b) At $(2, 9)$: $\text{MU}_X = 3$; $\text{MU}_Y = \frac{1}{2\sqrt{9}} = \frac{1}{6}$.

(c) As Y increases, $\text{MU}_Y = \frac{1}{2\sqrt{Y}}$ decreases (since $\sqrt{Y}$ increases in the denominator). This is **diminishing marginal utility**: each additional unit of Y adds less and less to total utility, which is a natural property of most goods.

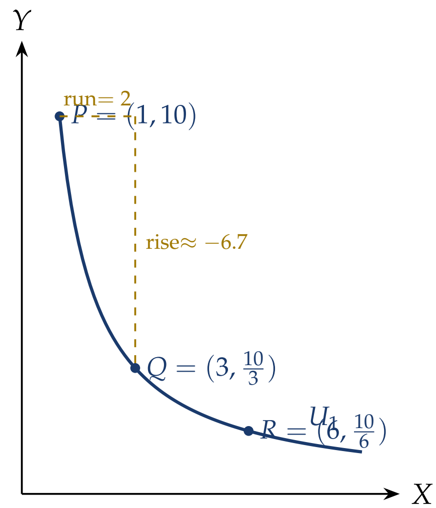
Solution to Q11

(a) From $P = (1, 10)$ to $Q = (3, 6)$: $\text{MRS} = \frac{6 - 10}{3 - 1} = \frac{-4}{2} = -2$. $|\text{MRS}_{PQ}| = 2$.

(b) From $Q = (3, 6)$ to $R = (6, 4)$: $\text{MRS} = \frac{4 - 6}{6 - 3} = \frac{-2}{3}$. $|\text{MRS}_{QR}| = \frac{2}{3}$.

(c) Yes, |MRS| fell from 2 to $\frac{2}{3}$ as we moved right. This is diminishing MRS. It implies the IC is convex (bowed inward toward the origin).

(d) Diagram: horizontal axis = X; vertical axis = Y. Plot a downward-sloping, convex curve. Mark $P = (1, 10)$, $Q = (3, 6)$, $R = (6, 4)$. From P to Q, draw a dashed horizontal segment (run = 2) and a vertical segment (rise = −4) meeting at $(3, 10)$. Label the MRS = −2. The curve flattens noticeably between Q and R.



Note: Using $U = XY = 10$ to fit $P = (1, 10)$. The points on this IC are $P = (1, 10)$, $Q = (3, 10/3 \approx 3.3)$, and $R = (6, 10/6 \approx 1.7)$. The key pattern—diminishing MRS—holds regardless of the exact coordinates.

Solution to Q12

(a) $U = X^{3/4}Y^{1/4}$: $\text{MU}_X = \dfrac{3}{4}X^{-1/4}Y^{1/4}$, $\text{MU}_Y = \dfrac{1}{4}X^{3/4}Y^{-3/4}$.

$$\text{MRS} = -\frac{\frac{3}{4}X^{-1/4}Y^{1/4}}{\frac{1}{4}X^{3/4}Y^{-3/4}} = -3 \cdot \frac{Y^{1/4+3/4}}{X^{3/4+1/4}} = -\frac{3Y}{X}.$$

(b) $U = X^{1/2} + Y^{1/2}$: $\text{MU}_X = \dfrac{1}{2}X^{-1/2}$, $\text{MU}_Y = \dfrac{1}{2}Y^{-1/2}$.

$$\text{MRS} = -\frac{\frac{1}{2}X^{-1/2}}{\frac{1}{2}Y^{-1/2}} = -\frac{Y^{1/2}}{X^{1/2}} = -\sqrt{\frac{Y}{X}}.$$

(c) $U = \ln X + 2\ln Y$: $\text{MU}_X = 1/X$, $\text{MU}_Y = 2/Y$.

$$\text{MRS} = -\frac{1/X}{2/Y} = -\frac{Y}{2X}.$$

(d) $U = 2X + 3Y$: $\text{MU}_X = 2$, $\text{MU}_Y = 3$.

$$\text{MRS} = -\frac{2}{3}.$$

The MRS is **constant**—it does not depend on X or Y. This means the IC is a straight line with slope $-2/3$. Goods X and Y are **perfect substitutes**: the consumer always trades 2 units of X for 3 units of Y at the same rate, regardless of how much of each they have.

Solution to Q13

(a) Since $U_2 > U_1$, any bundle on U_2 is preferred to any bundle on U_1. So $B \succ A$. Bundle Z is on both curves simultaneously, so $Z \sim B$ (because Z is on U_2) and $Z \sim A$ (because Z is on U_1).

(b) From above: $B \succ A$, $Z \sim B$, and $Z \sim A$. By transitivity applied to $Z \sim B$ and $B \succ A$: we should have $Z \succ A$. But we also said $Z \sim A$—a contradiction. Hence transitivity is violated.

(c) Indifference curves can never cross because doing so would violate the transitivity of preferences, which is a fundamental assumption of rational consumer behavior.

Solution to Q14

(a) Set $Q^d = Q^s$:

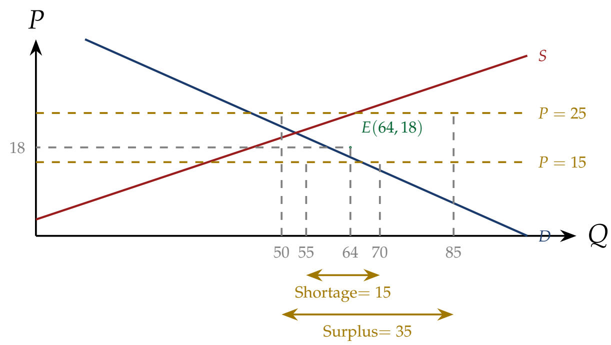
$$100 - 2P = 10 + 3P \implies 90 = 5P \implies P^* = 18.$$

$Q^* = 100 - 2(18) = 64$. Equilibrium: $P^* = 18$, $Q^* = 64$.

(b) At $P = 15$: $Q^d = 100 - 30 = 70$; $Q^s = 10 + 45 = 55$. Excess demand $= 70 - 55 = 15$. There is a **shortage** of 15 units.

(c) At $P = 25$: $Q^d = 100 - 50 = 50$; $Q^s = 10 + 75 = 85$. Excess supply $= 85 - 50 = 35$. There is a **surplus** of 35 units.

(d) Diagram: horizontal axis $= Q$; vertical axis $= P$. Plot a downward-sloping demand curve D and an upward-sloping supply curve S. Mark equilibrium E at $(64, 18)$. Mark a horizontal dashed line at $P = 15$ (below E), indicating excess demand. Mark a horizontal dashed line at $P = 25$ (above E), indicating excess supply. At $P = 15$, price will rise toward P^*; at $P = 25$, price will fall toward P^*.



Solution to Q15

$U = X^{2/3}Y^{1/3}$.

(a) $\text{MU}_X = \frac{2}{3}X^{-1/3}Y^{1/3}$; $\text{MU}_Y = \frac{1}{3}X^{2/3}Y^{-2/3}$.

(b) $\text{MRS} = -\frac{\text{MU}_X}{\text{MU}_Y} = -\frac{\frac{2}{3}X^{-1/3}Y^{1/3}}{\frac{1}{3}X^{2/3}Y^{-2/3}} = -2 \cdot \frac{Y^{1/3+2/3}}{X^{2/3+1/3}} = -\frac{2Y}{X}$.

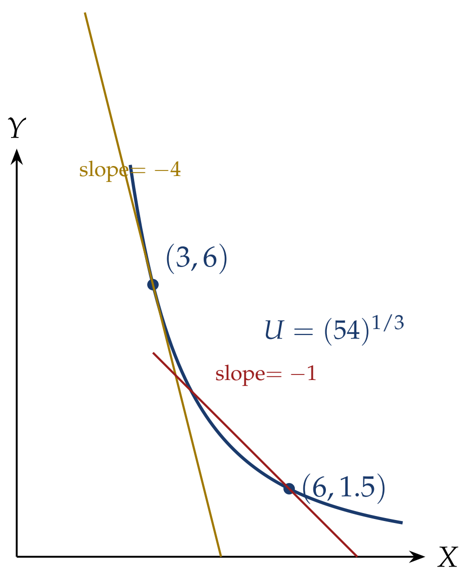
(c) At $(3,6)$: $\text{MRS} = -\frac{2(6)}{3} = -4$. Consumer gives up 4 units of Y per unit of X.

At $(6,3)$: $\text{MRS} = -\frac{2(3)}{6} = -1$. Consumer gives up only 1 unit of Y per unit of X.

At $(3,6)$ the consumer has relatively little X and lots of Y, so X is precious and they are willing to give up a lot of Y for it. At $(6,3)$ the situation reverses.

(d) The slope of the IC is $\text{MRS} = -4$ at $(3,6)$, confirmed by the formula.

(e) As X increases along the IC (with Y falling), $\text{MRS} = -2Y/X$ decreases in magnitude. For example, moving from $(3,6)$ to $(6,3)$, $|\text{MRS}|$ goes from 4 to 1.



Solution to Q16

(a) **False.** The slope of the IC equals $-\text{MU}_X/\text{MU}_Y$. A steeper IC means a larger $|\text{MRS}| = \text{MU}_X/\text{MU}_Y$, which means MU_X is large relative to MU_Y. So the consumer values X relatively more (not Y). A steeper IC means the consumer is willing to give up a lot of Y per unit of X—which reflects that X is highly valued, not Y.

(b) **True.** If one good is an "economic bad" (say the consumer dislikes good Y), then $\text{MU}_Y < 0$. In that case, $\text{MRS} = -\text{MU}_X/\text{MU}_Y > 0$ (since $\text{MU}_X > 0$ and $\text{MU}_Y < 0$). A positive MRS means the IC is upward sloping. Example: $U = X - Y^2$ with Y representing pollution—more pollution reduces utility, so the IC slopes upward.

(c) For $U = X^a Y^{1-a}$, the MRS is $-\dfrac{aY}{(1-a)X}$. For the consumer to be indifferent between $(1, 8)$ and $(4, 2)$, they must lie on the same IC:

$$1^a \cdot 8^{1-a} = 4^a \cdot 2^{1-a}.$$

Taking logarithms:

$$(1-a)\ln 8 = a\ln 4 + (1-a)\ln 2.$$

$$(1-a)(\ln 8 - \ln 2) = a\ln 4 \implies (1-a)\ln 4 = a\ln 4 \implies 1 - a = a \implies a = \tfrac{1}{2}.$$

So $U = X^{1/2}Y^{1/2} = \sqrt{XY}$. Check: $\sqrt{1 \times 8} = \sqrt{8}$ and $\sqrt{4 \times 2} = \sqrt{8}$.✓

Solution to Q17 (Data-Based: Bundle Ranking)

(a) Computing $U = XY$ for each bundle:

Bundle	X	Y	$U = XY$
A	2	8	16
B	4	4	16
C	1	12	12
D	6	2	12
E	3	6	18

Ranking: $E(18) > A = B(16) > C = D(12)$.

(b) Bundles A and B lie on the same IC ($U = 16$). Bundles C and D lie on the same IC ($U = 12$). They give equal utility so the consumer is indifferent between them.

(c) The student is incorrect. Bundle $A = (2, 8)$ has more Y than $C = (1, 12)$? No — C has *more* Y (12 vs 8) but less X (1 vs 2). The utility values are $U_A = 16 > 12 = U_C$, so A is preferred to C despite C having more Y. Monotonicity requires *more of both* to guarantee higher utility; having more of one good is not sufficient.

(d) Bundle $B = (4, 4)$ is the most balanced (equal quantities). MRS at B: for $U = XY$, $\text{MU}_X = Y = 4$ and $\text{MU}_Y = X = 4$, so $\text{MRS} = -\text{MU}_X/\text{MU}_Y = -4/4 = -1$. This means at B the consumer is willing to trade one unit of X for exactly one unit of Y — the goods are equally valued at this bundle.

(e) *(Week 2 preview)* Budget: $3X + 2Y = 20$. Check affordability: A: $3(2) + 2(8) = 22 > 20$ (not affordable). B: $3(4) + 2(4) = 20$ (affordable, on the line). C: $3(1) + 2(12) = 27 > 20$ (not affordable). D: $3(6) + 2(2) = 22 > 20$ (not affordable). E: $3(3) + 2(6) = 21 > 20$ (not affordable). Only B is exactly affordable. The optimal bundle requires applying the tangency condition from Week 2.

Solution to Q18 (Explain to a Friend — Level 2)

A good answer addresses: utility numbers are **ordinal**, not cardinal. They tell us the *ranking* of bundles but not by how much one is preferred to another. If $U = 15$ from 3 slices and $U = 5$ from 1 slice, all we can conclude is that 3 slices are preferred to 1 slice. We cannot say the consumer is "three times as happy" — that would require cardinal utility, which economists cannot measure. The numbers 15 and 5 could equally well be replaced by 100 and 2, as long as the ranking is preserved.

Solution to Q20 (formerly Q17: Demand, Supply, Simultaneous Shifts)

New supply: $Q^s = -160 + 3P$. Set equal to original demand:

$$500 - 2P = -160 + 3P \implies 660 = 5P \implies P^{**} = 132. \quad Q^{**} = 500 - 264 = 236.$$

Price rose from 120 to 132; quantity fell from 260 to 236.

Both shifts: $Q^d = 560 - 2P$ and $Q^s = -160 + 3P$:

$$560 - 2P = -160 + 3P \implies 720 = 5P \implies P^{***} = 144. \quad Q^{***} = 560 - 288 = 272.$$

Price rose significantly (from 120 to 144); quantity rose slightly (from 260 to 272) because the demand increase partially offset the supply reduction.

Diagram: plot original D and S crossing at $(260, 120)$. Shift S left to S' (crossing original D at $(236, 132)$). Then shift D right to D' (crossing S' at $(272, 144)$). Label all three equilibria. Show arrows indicating direction of shifts.

Solution to Q18

$U = XY + 2X$.

(a) $\text{MU}_X = Y + 2; \quad \text{MU}_Y = X$.

(b) $\text{MRS} = -\dfrac{Y+2}{X}$. This depends on both X and Y, so it is **not constant**. The ICs are therefore **curved** (not straight lines), reflecting diminishing MRS.

(c) IC where $U = 20$: $(X)(Y) + 2X = 20 \implies X(Y+2) = 20 \implies Y = \dfrac{20}{X} - 2$.

At $X = 2$: $Y = \dfrac{20}{2} - 2 = 8$. Bundle: $(2, 8)$.

At $X = 4$: $Y = \dfrac{20}{4} - 2 = 3$. Bundle: $(4, 3)$.

(d) MRS at $(2, 8)$: $-\frac{8+2}{2} = -5$.

MRS at $(4, 3)$: $-\frac{3+2}{4} = -\frac{5}{4} = -1.25$.

$|\text{MRS}|$ fell from 5 to 1.25 as X increased. MRS is **diminishing** in absolute value.✓

Solution to Q19

(a) For a consumer who derives positive utility from both goods (monotonicity holds), both $\text{MU}_X > 0$ and $\text{MU}_Y > 0$. The slope of the IC equals $-\text{MU}_X/\text{MU}_Y < 0$. A negative slope means the IC runs downward from left to right: if you gain more X, you must lose some Y to stay at the same utility level.

(b) A convex (bowed-inward) IC reflects diminishing MRS. As you accumulate more X and less Y along an IC, the slope becomes less steep in absolute value—the curve flattens. This means the consumer is willing to give up less and less Y per additional X, which is rational: when X is abundant and Y is scarce, the marginal unit of X is worth less while the marginal unit of Y is more precious. As a consequence, the balanced bundle between two extreme bundles on the same IC lies above the IC and hence yields higher utility. Balanced consumption is preferred.

(c) The MRS tells us the consumer's internal valuation of X relative to Y—how many units of Y they are willing to trade for one unit of X. Market prices tell us the external trade-off: how many units of Y can be obtained in exchange for one unit of X at prevailing prices, i.e., P_X/P_Y. Later in the course, we will show that a consumer maximizes utility when $|\text{MRS}| = P_X/P_Y$—when the internal trade-off rate exactly equals the market trade-off rate. If they differ, the consumer can rearrange their spending to increase utility.

Solution to Q20

$F(L, K) = L^{1/2}K^{1/2}$.

(a) $MP_L = \frac{\partial F}{\partial L} = \frac{1}{2}L^{-1/2}K^{1/2} = \frac{\sqrt{K}}{2\sqrt{L}}$.

(b) $MP_K = \frac{\partial F}{\partial K} = \frac{1}{2}L^{1/2}K^{-1/2} = \frac{\sqrt{L}}{2\sqrt{K}}$.

(c) As L increases (holding K fixed), $\sqrt{L}$ increases, so $MP_L = \frac{\sqrt{K}}{2\sqrt{L}}$ decreases. This is **diminishing marginal returns to labor**: each additional worker contributes less to output when capital is fixed.

(d) $\text{MRTS} = -\frac{MP_L}{MP_K} = -\frac{\frac{\sqrt{K}}{2\sqrt{L}}}{\frac{\sqrt{L}}{2\sqrt{K}}} = -\frac{K}{L}$.

The MRTS is the amount of capital the firm can give up when hiring one more unit of labor, while keeping output constant. It is the slope of the firm's isoquant—the production analog of the consumer's indifference curve.

(e) At $(L, K) = (4, 9)$: MRTS $= -9/4 = -2.25$. The firm can reduce capital by 2.25 units per additional worker.

At $(L, K) = (9, 4)$: MRTS $= -4/9 \approx -0.44$. When labor is abundant and capital is scarce, substituting capital for labor becomes much harder.

Solution to Q21

$U(X, Y) = (X + 1)(Y + 2)$.

(a) $\text{MU}_X = \frac{\partial}{\partial X}[(X+1)(Y+2)] = Y + 2$. $\quad \text{MU}_Y = \frac{\partial}{\partial Y}[(X+1)(Y+2)] = X + 1$.

(b) $\text{MRS} = -\frac{\text{MU}_X}{\text{MU}_Y} = -\frac{Y+2}{X+1}$. Not constant, so ICs are curved.

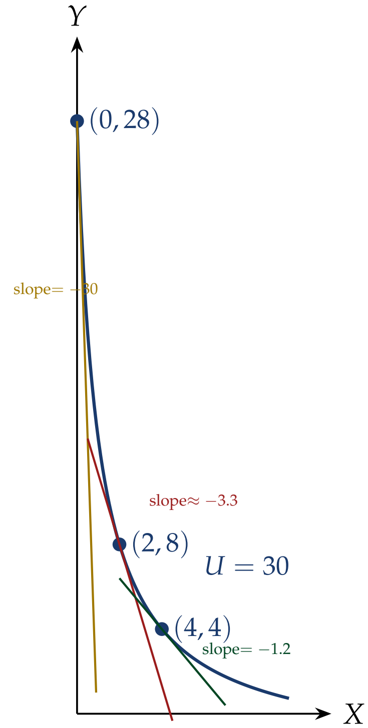
(c) $U = 30$: $(X+1)(Y+2) = 30 \implies Y + 2 = \frac{30}{X+1} \implies Y = \frac{30}{X+1} - 2$.

(d) $X = 0$: $Y = 30/1 - 2 = 28$. Bundle: $(0, 28)$. $X = 2$: $Y = 30/3 - 2 = 8$. Bundle: $(2, 8)$. $X = 4$: $Y = 30/5 - 2 = 4$. Bundle: $(4, 4)$.

(e) MRS at each bundle:

- $(0, 28)$: MRS $= -(28+2)/(0+1) = -30$.
- $(2, 8)$: MRS $= -(8+2)/(2+1) = -10/3 \approx -3.33$.
- $(4, 4)$: MRS $= -(4+2)/(4+1) = -6/5 = -1.2$.

$|\text{MRS}|$ decreases: $30 > 3.33 > 1.2$. **MRS is diminishing.**✓

(f) 

(g) At bundle $(2, 8)$, the MRS $= -10/3 \approx -3.33$. This tells us the consumer is willing to give up approximately 3.33 units of Y in exchange for one additional unit of X, while remaining on the same IC. In other words, the consumer internally values 1 unit of X as equivalent to 3.33 units of Y. If the market offers X for fewer than 3.33 units of Y (i.e., the price ratio $P_X/P_Y < 3.33$), the consumer would benefit from buying more X and less Y.

Solution to Q22

(a) Economic profit = Revenue − Explicit costs − Opportunity cost. Revenue = \$90,000. Explicit costs = \$50,000. Opportunity cost = \$120,000 (foregone salary).

$$\text{Economic profit} = 90{,}000 - 50{,}000 - 120{,}000 = -\$80{,}000.$$

(b) Based on economic profit alone, she should **not** start the company. The startup makes her worse off by \$80,000 compared to staying at her job. Accounting profit (\$40,000) looks positive, but it ignores the foregone salary.

(c) Non-monetary factors that might still influence her decision include: the value of being her own boss (autonomy), the potential for the company to grow substantially in future years, personal passion for the product, risk tolerance, and non-financial satisfaction from entrepreneurship.

Solution to Q23

The student's reasoning is incorrect because utility is **ordinal**, not cardinal. Ordinal means utility numbers only rank outcomes—they do not measure "how much more" satisfaction one bundle provides over another. The statement $U = 15$ versus $U = 5$ tells us only that 3 slices is *preferred to* 1 slice. It does not mean 3 slices yields three times the satisfaction. We could equally well represent the same preferences with $U' = 100$ and $U' = 10$, or $U' = 1000$ and $U' = 999$—as long as the ranking is preserved. Any positive monotonic transformation of a utility function represents the same preferences.

Solution to Q24

(a) A steep IC means a large |MRS|. Since $\text{MRS} = -\text{MU}_X/\text{MU}_Y$, a large |MRS| means MU_X is large relative to MU_Y—Consumer A places high value on coffee relative to tea. At the margin, consumer A is willing to give up a lot of tea to get one more unit of coffee.

(b) A flat IC means a small |MRS|. Consumer B is only willing to give up a little tea for an extra unit of coffee—they value the two goods more similarly, or place relatively more value on tea.

(c) Consumer A, with steep ICs and a high |MRS|, values coffee much more than tea. If the market price ratio does not fully reflect this strong preference, consumer A will tend to buy mostly coffee and very little tea—a corner or near-corner solution. Consumer B, with flat ICs and a low |MRS|, is more willing to substitute and will likely consume a more balanced mix of both goods.

Solution to Q25

(a) Policy 1 (a tax) raises the price of cigarettes. This is a change in price, so it causes a *movement along* the existing demand curve—it does not shift the curve. Supply is unaffected (assuming the tax is on consumers). Equilibrium price rises and quantity falls.

Policy 2 (a public health campaign) changes consumers' tastes—it reduces the desire for cigarettes at any given price. This shifts the demand curve *leftward*.

(b) A movement along the demand curve occurs when only the price of the good changes, holding all other factors constant. A shift of the demand curve occurs when a non-price factor (income, tastes, prices of related goods, expectations) changes—the entire relationship between price and quantity demanded changes.

(c) If demand is very inelastic, consumers barely reduce their purchases when price rises (Policy 1 has a small effect on quantity). Policy 2, which directly reduces desire for cigarettes, is more effective because it shifts the entire demand curve leftward—reducing quantity at every price level.

Solution to Q26

(a) No, the consumer is **not** at his optimum. His $|\text{MRS}| = 3$ means he values the next unit of X at 3 units of Y internally. But the market only asks him to give up $P_X/P_Y = 2$ units of Y for one unit of X. He can get X at a "cheaper" rate in the market than he would need to remain indifferent—so he should buy more X.

(b) He should buy **more** X and less Y. Formally: $\text{MU}_X/P_X > \text{MU}_Y/P_Y$ at his current bundle (the bang-per-buck from X exceeds that from Y), so reallocating spending toward X increases utility.

(c) At the optimum, the consumer's internal trade-off rate equals the market trade-off rate:

$$|\text{MRS}| = \frac{P_X}{P_Y} \iff \frac{\text{MU}_X}{P_X} = \frac{\text{MU}_Y}{P_Y}.$$

This is the tangency condition between the budget line and the highest attainable indifference curve—the formal proof is developed in Week 2.

Solution to Q26 (Explain to a Friend — Level 3)

A strong answer uses an everyday example and avoids jargon. For instance: *"Imagine you are at a buffet choosing between pizza and pasta. Right now you have a lot of pasta and very little pizza. The MRS tells you how many slices of pasta you would be willing to give up for one more slice of pizza and still feel just as satisfied. When you have lots of pasta and little pizza, you are willing to give up many slices of pasta for just one pizza slice — that is a high MRS. But as you eat more pizza and less pasta, each additional pizza slice is less exciting and each pasta slice you give up hurts more — so the MRS falls. That is diminishing MRS."* The key insight is that the MRS is not fixed: it changes depending on where you are on the indifference curve, which is why the curve bows inward rather than being a straight line.

End of Week 1 Notes

Intermediate Microeconomics — Week 1 — Prof. Naveen Sunder